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DYNAMIC SOIL-STRUCTURE INTERACTION : RECENT DEVELOPMENTS AND SIMPLIFIED DESIGN

ABSTRACT : In this paper, we present recent developments and discuss about the available results concerning the dynamic soil-structure interaction. It will be the start point to analyse the possibilities for the practical engineer to solve simple problems. In the same time, some explanations on the evolution of the papers content are proposed. At last, we give informations on errors introduced by simplified or equivalent methods and some new results concerning strip footings on a layer resting on an inclined substratum.

1 INTRODUCTION

Concerning dynamic sol-structure interaction, a lot of papers is published each year. The reading of these papers shows clearly an evolution of their contents : there are less and less numerical results or empirical relations obtained from systematic parametric calculations. Looked at from this point of view, the most important contributions were published by Dominguez and Roesset (1978) for rectangular footings and by Gazetas (1983) who has written a very complete State of the Art. The author (1992) himself completed this work in giving all known impedance functions in a practical form. Wolf (1994) proposed recently a new very detailed approach to calculate impedance functions. During the ten last years, the publications describe new theoretical approaches or improve existing numerical methods. Generally, authors illustrate their methods with the help of one example and/or a comparison with results obtained by other authors. Consequently, the practical engineer has only the results for few cases.

This statement brings to put the question if an engineer can practically use these informations to solve his problem. In the background of this discussion is also the question - very important for all teachers - of the contents of the educational program of our students.

2 PRESENT SITUATION

2.1 *Recent developments*

We give here two examples to illustrate recent new developments concerning the dynamic soil-structure interaction.

2.1.1 Non deterministic approach

The first example concerns the non deterministic approach presented by Toubalem & Labbé (1996). In order to study experimentaly the soil-structure interaction, dynamic tests were performed on a nuclear reactor scale model on the HUALIEN experimental site in Taïwan.

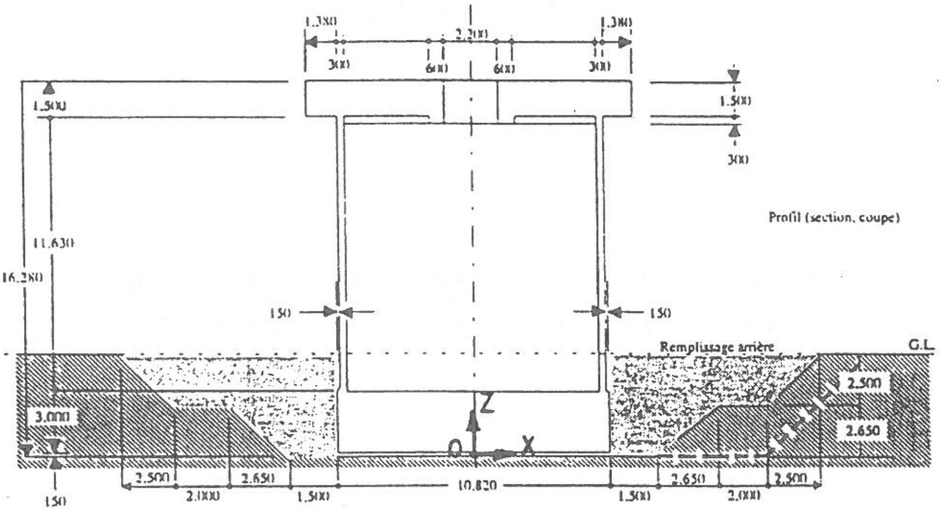

fig.1. Map of the HUALIEN scale model (after Billet et al)

The figure 1 presents the mape of this scale model. It is an axisymetric structure with a diameter about 11 m and an height about 16 m. The soil foundation can be considered as homogeneous and isotropic, so that the behaviour of the soil-structure system should be axisymetric. Dynamic tests were performed to measure the eigenfrequencies of the soil-structure system. It was expected that the eigenfrequencies for the rocking movements are the same about two perpendicular directions. In fact, the tests give two values : 4.1 and 4.6 Hz. It corresponds to a relativ difference about 10%.

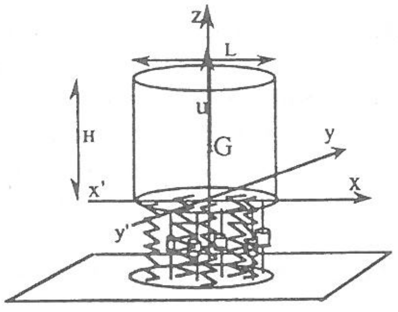

fig.2. Representation of a bidimensional Winkler model (after Toubalem et al)

In order to explain this difference, Toubalem et al take into account the stochastic character of soil. They consider a rigid cylindrical structure supported by a two-dimensional continuum of springs (Winkler model) characteristics of which are uncertain (fig.2). The stiffness of the springs is written as a constant value K_0 and a function ΔK (x, y) with a mean value of 0. The figure 3 gives an example of the possible fluctuation of the stiffness of the springs. The authors calculated the eigenfrequencies corresponding to this stochastic variation of the stiffness.

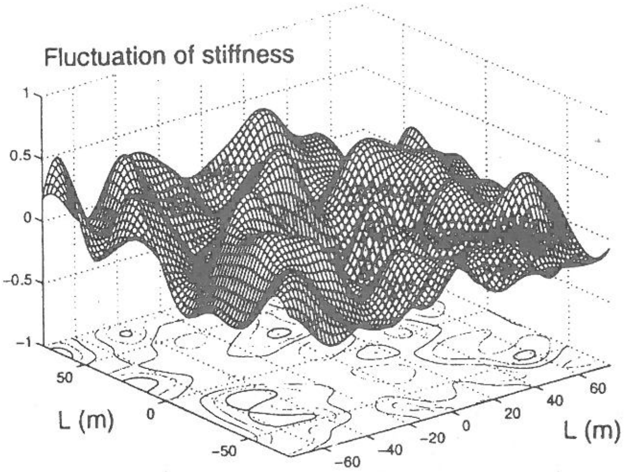

fig.3.Example of realisation of random fluctuations associated with rigidity (afer Toubalem et al)

Figure 4 shows that it exists a large probability to obtain two eigenfrequencies with a difference about 1 or 2 percent. The authors note also that the mean value of these two frequencies is smaller as the eigen-frequency obtained in considering only constant stiffness of the springs.

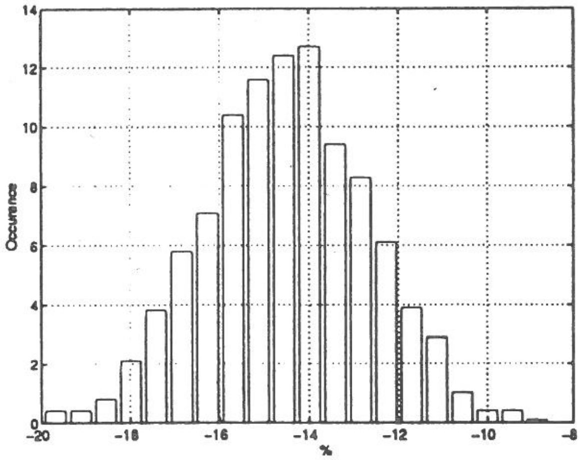

fig.4. Histogram of the difference of rocking eigenvalues (after Toubalem et al)

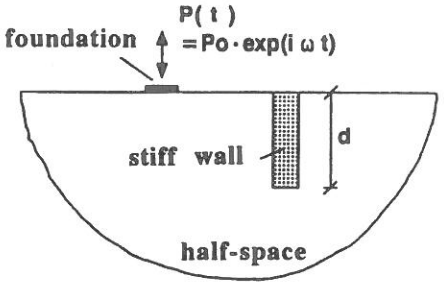

fig.5. Stiff wall as a vibration barrier

2.1.2 Ground vibration isolation

The second example concerns the development of a new technics of ground vibration isolation . In order to reduce the vibrations due to a surface source, one method consists to introduce into the soil a stiff wall which is a barrier for the wave propagation (fig.5). An other possibility to reduce vibrations consists in preventing wave in introducing an obstacle under the source as presented by Chouw, Le & Schmid (1991).
In order to explain the principle, we consider in a first step a footing (source) on a layer resting on a substratum.

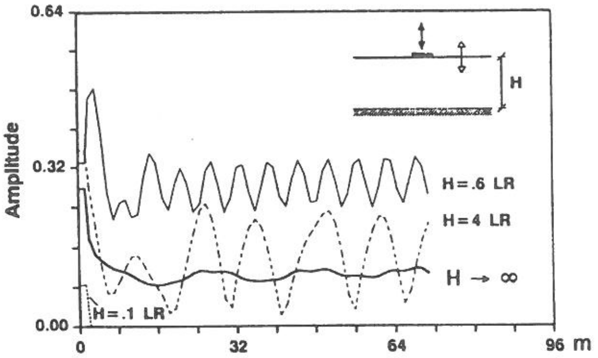

fig.6. Amplitude of vertical dispacement at surface of soil versus distance from the source (after Chouw et al)

The figure 6 shows the amplitude of the vertical displacements at the surface of the soil versus the distance from the source. It is well know that in this case the layer has eigenfrequencies. The figure shows that :
- for a thickness of the layer of 4 LR (length of the Rayleigh'wave), we can have greater or smaller amplitude as in the case of a half-space,
- for a thickness of the layer of 0.6 LR, the amplitude is ever greater as in the case of a half-space,
- for a very small thickness of the layer - for example 0.1 LR -, we have no movement of the soil. It means that in this case, the waves cannot exist.

The idea developped by Schmid consists to introduce under the source and near the source a rigid block which prevent the existing of wave as in the case of a bedrock. The figures 7 show the results obtained with this wave barrier. For an obstacle in concrete, the ratio of the velocities of the shear waves is very large (Cs2 / Cs1 > 12) , so that we can conclud that this method can be very efficient.

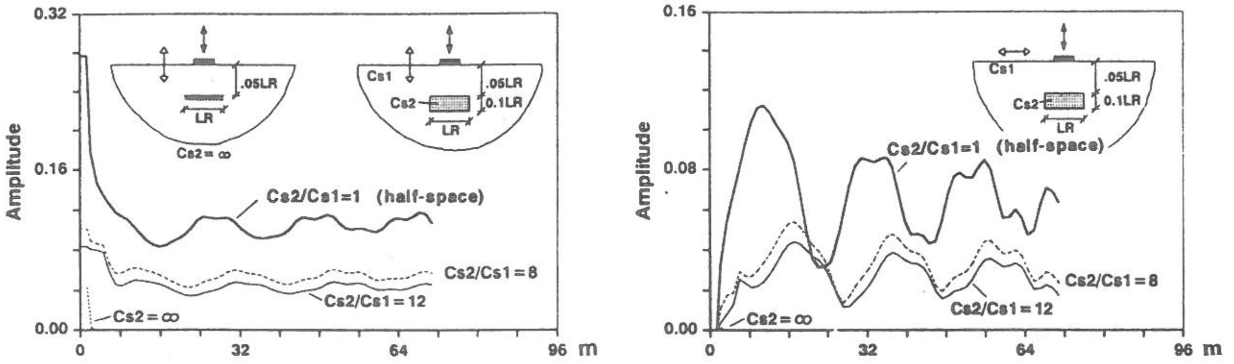

fig.7. Amplitude of vertical and horizontal displacements at the surface of soil versus distance from the source (after Chouw et al)

On figure 8, we can compare the effectivness of both methods and see that it is more efficient to place the obstacle under the source as at its side of the source : the amplitudes are effectively reduced on all the surface in the first case, whereas the wall has no effectiveness at the source and more generally at its left side.

These results were obtained in using the Boundary Elements Methods with the software SSI developed by Schmid.

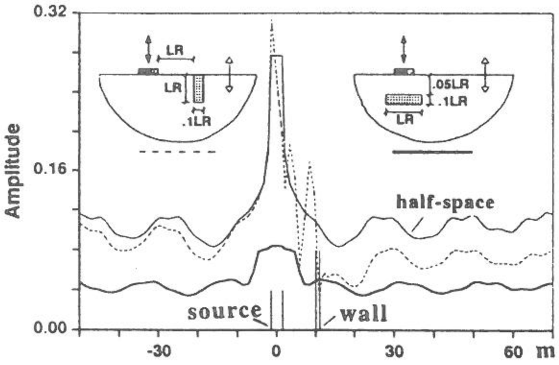

fig.8. Comparison of the screening effectivness (after Chouw et al)

2.2 *Available results*

Before we indicate the cases for which there is significant lot of results, we recall that dynamic soil-structure interaction is usually discribed by the *Impedance Functions* which are the complex dynamic stiffnesses K of the soil-footing system. Impedance functions are every calculated for massless footings. This permits the determination of the movements for any mass of the footing-structure system. The real part describes the stiffness without energy loss, and the imaginary part the energy loss (damping). These stiffnesses must be estimated for each degree of freedom of the foundation. The general form is :

$$K = K^R + i\,K^I = \frac{P\,(t)}{u\,(t)} \qquad (1)$$

where u(t) is the time dependent displacement (or rotation), and P(t) the time dependent exciting force (or moment).

The following notations will be used :

V : vertical translation,
H_x : horizontal translation in x-direction,
H_y : horizontal translation in y-direction,
R_x : rotation (rocking) about x-axis,
R_y : rotation (rocking) about y-axis,
T : rotation (torsion) about vertical axis.

For each degree of freedom, dynamic stiffness depends on so many parameters, for example :

- soil caracteristics : shear modulus G, Poison's ratio ν, density ρ,
- soil and foundation geometry,
- excitation frequency.

Frequency is usually defined by the dimensionless circular frequency a_0 :

$$a_0 = \frac{\omega r}{c_s} \tag{2}$$

in which : ω is the circular frequency, r a characteristic dimension of the foundation, c_S the shear wave velocity in the soil.

We can also note that the static stiffness is the value of the real part of the impedance function for $a_0 = 0$.

Available results concern circular footings, rectangular footings, and strip footings.

Circular footings

Due to its symmetry, the theoretical solution of circular foundation is easier to obtain than that of rectangular foundation which needs tridimensional calculations. The first results were presented by Reissner in 1936, therefore before the development of computers.

We dispose on very complete results for :

- footing on a half-space medium (with or without embedment depth),
- footing on a layer resting on an horizontal substratum (with or without embedment depth),
- footing on an horizontal layer resting on an half-space medium (without embedment depth).

For these cases, it exists empirical relations - available only in a limited range of validity - giving the static stiffnesses.

Rectangular footings

It is the more classical geometry for a foundation. But significant results concern only foundations on a half-space medium, with or without embedment depth.

Strip footings

Available results concern :

- footing on a half-space medium (without embedment depth),
- footing on a layer resting on an horizontal substratum (with or without embedment depth),
- footing on an horizontal layer resting on an half-space medium (without embedment depth).

Every engineer in geotechnical engineering knows that :

- the more classical situation of a footing soil is a layer on an rigid substratum,
- the more used foundation is the rectangular foundation,
- in practice, a footing has ever an embedment depth corresponding to soil freezing depth (between 0.6 and 1 meter depending on geographical situation and climatic conditions).

We must state that we have not at disposal any result concerning this case! In addition, if no mistake, there are no results concerning an inclined substratum however is the geometrical form of the footing.

2.3 *Reasons of present situation*

Softwares able to calculate cases as described previously exist. What explanations of recent new complete results non-publishing (tables or charts) as it was the practice in the sixties and in the seventies ?

The first reason is certainly the expansion of computers and softwares as explained previously. We note the same evolution in other domains as Soil Mechanics.The most cases described on

§ 2.2 were calculated and presented in Ph.D thesis. It appears that systematic calculations are now not considered as sufficient level for Ph.D works. We can easily agree this opinion.
It is possible that the majority of authors thinks each engineer possess his own necessary hardware and software - what it is true - and is able to develop himself his softwares from the indications given in the publications - what it is not so easy because the practical engineers have not necessarily the indispensable long experience for the use of these sofwares, especially in dynamic problems. By virtue of this principle, everybody should be able to calculate himself the necessary values to solve his problem. Looked at from this point of view, publication of systematic results is not necessary. Of course, this way must be used to solve complex problems concerning constructions as earthdams or nuclear power installations which present a risk for population and environment : simplified solutions with large approximations are not acceptable in these cases. But often, the engineer needs immediately results to justify the feasability of his project. There is every reason to suppose he will prefer use published values instead to do again complicated calculations which demands a very good knowledge of the software, and takes time. Only this last point will be discussed here.

The second reason is the number of tables or charts needed to present complete results on new cases because increasing of number of parameters. For example, the impedance functions of a rectangular footing on an layer resting on an inclined substratum depend on 10 parameters :

- footing parameters : length, width, depth of embedment, length of lateral welded contact beetwen soil and foundation,
- soil parameters : thickness of layer, slope of substratum, shear modulus, Poison's ration, internal damping,

and of course of the frequency.
Using dimensionless values, these 10 parameters can be reduced to 8. Supposing each parameter can take 5 values, 5^6 charts giving one stiffness as frequency function and containing 5 curves are necessary. We have for each stiffness a real part an a imaginary part, 6 degrees of freedom and 2 coupled translation-rotation modes. This demands 5^6 x 2 x 8 = 250,000 charts ! This difficulty can be resolved in giving the results on floppy-disks or CD-ROM.
But it is obvious that it is not possible to publish all these results in a Journal or in the Proceedings of a Congress. It will be also necessary that papers' authors' instructions must permit to publish large charts in ordre to be able to easy read values from these charts.

3 ERRORS INDUCED BY APPROXIMATE METHODS

If the engineer does not find directly the response of his problem in the literature, the risk is that the engineer will try to solve his problem by comparison to typical well-known cases without knowning the induced error. We present yet any example to illustrate this.

3.1 *Error induced by equivalent method*

The typical example concerns the rectangular foundation replaced by an equivalent circular footing for which there are so many results at disposal as explain previously.
To show the error induced by using the equivalent circular footing method, we compare in a first step the results obtained directly by Dominguez et al (1978) for a square footing, and those obtained by using an equivalent circular footing. The radius is calculated by writing that it has respectively the same area for translation movements and same moment of inertia for rotation movements.
Relative errors are given in table 1. B is the half width of the foundation, D the embedment

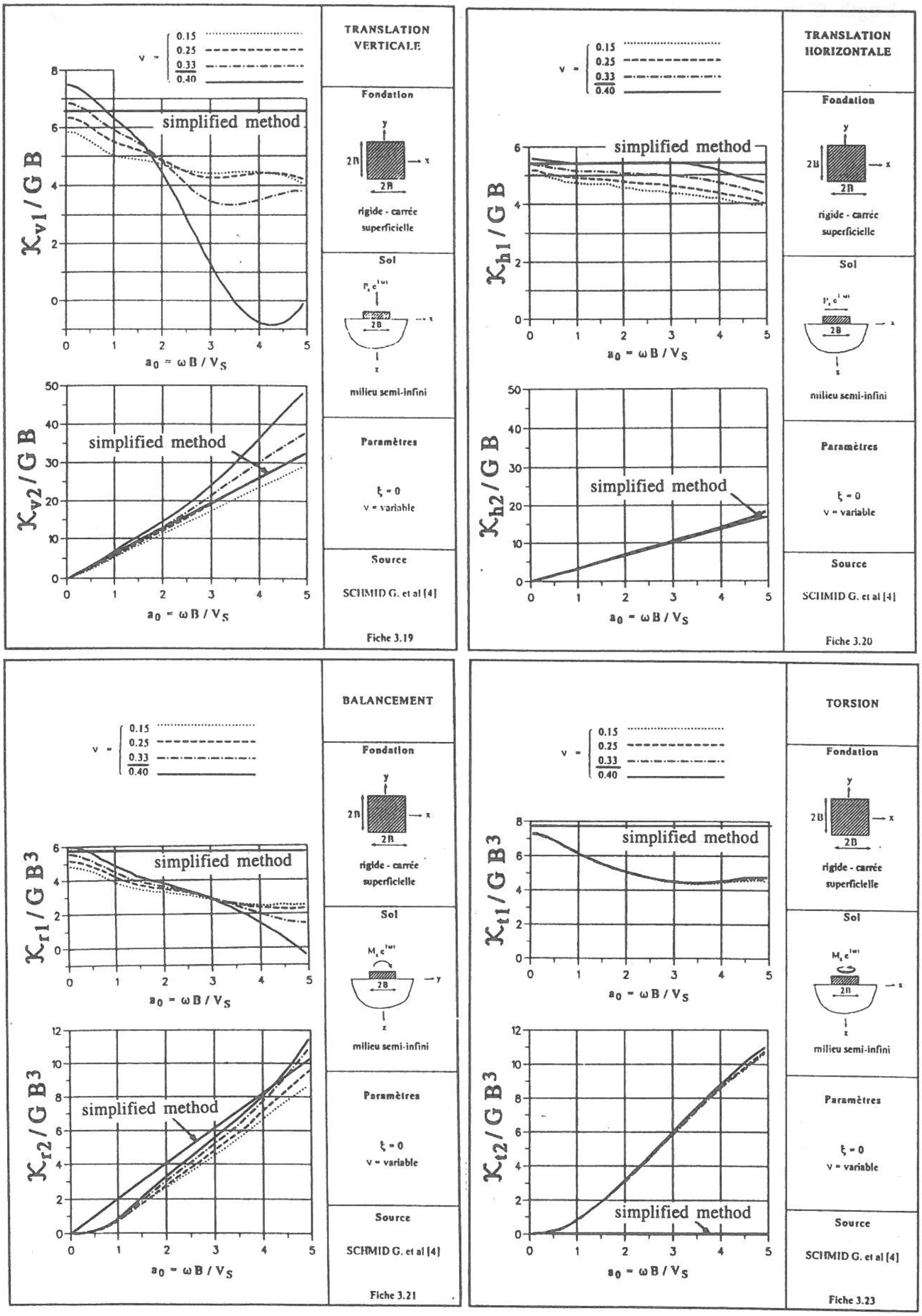

fig.9. Comparison complete functions - simplified method

length, G the shear modulus of the soil, and ν its Poisson'ratio.

Embedment	Mode				
D/B	Vertical	Horizontal	Rocking	Horiz.-Rock.	Torsion
0	-5.05	-4.04	-9.36	0	-8.92
0.5	-5.34	-18.54	-4.12	-14.98	-16.01
1	-3.85	-22.55	-19,82	-25.62	-15.74
1.5	-0.93	-22.53	-34.44	-31.10	-13.98
2	2.54	-20.12	-45.45	-33.85	-11.46

Table 1. Square footing - static stiffnesses (D/B) - relative errors (%)

This table shows clearly the limits of this equivalent method concerning the static stiffnesses, especially for horizontal and rocking movements which are induced by wind, earthquakes, and so on. However, one has at one's disposal only few results concerning rectangular footings on a half-space soil foundation. For embedded footings, Dominguez gives only results for two ratios length / width = 1 and 2. For surface footings without embedment depth, one can find more results.

An other approximation consists to replace the dynamic stiffnesses which are functions of the frequecncy by constant values. The figures 9 show the comparison between the approximate stiffnesses and the complete functions for square footing without embedment and $\nu = 0.33$. On this charts, the index 1 refers to the real part and the index 2 the imaginary part. In the same way, v_S is the velocity of the shear waves in the soil. We can see that it exists important differencies for the real parts of the stiffnesses. The imaginary parts are better described except for the torsion movement.

3.2 *Error induced by negligting coupling*

On the other hand, it is also necessary to give all terms of the impedance matrix. Often are given only the terms of the main diagonal which concern the uncoupled movements. It is well know that horizontal translation and rocking movements are ever coupled. Of course, the values taken by the coupling terms are minor for a non embedded surface footing. In this case, it is not possible to neglect coupled horizontal displacement - rocking term, all the more as footing are generaly loaded by inclined and eccentric forces.To illustrate this purpose, we come back to the example of the square footing with an embedment ratio D/B = 1/2, considering here also only static aspects of the problem (fig.10).

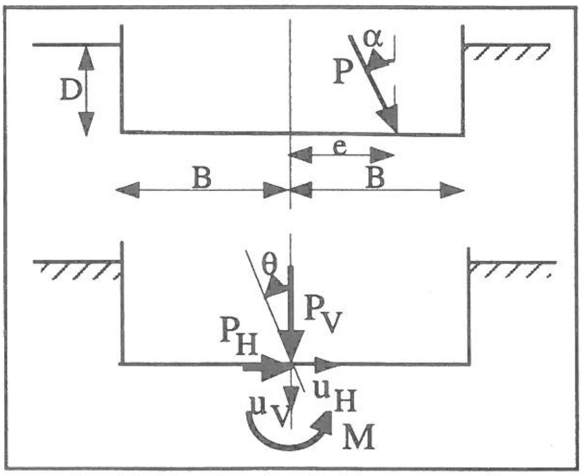

fig.10. Footing loaded by eccentric inclined force

Equations (3) give the transformation of the eccentric inclined load P to vertical load P_V, horizontal load P_H and moment M applied on the center of the base of the fondation.

$$\begin{aligned} P_V &= P\cos\alpha \\ P_H &= P\sin\alpha \\ M &= -\,e\,P\sin\alpha \end{aligned} \qquad (3)$$

Vertical u_V and horizontal u_H displacements, and rotation θ can be calculated by using equations (4).

$$\begin{aligned} u_V &= \frac{P_V}{K_V} \\ u_H &= \frac{K_H P_H - P_{HR} M}{K_H K_R - K_{HR}^2} \\ \theta &= \frac{K_H M - K_{HR} P_H}{K_H K_R - K_{HR}^2} \end{aligned} \qquad (4)$$

Figures 11 show that the error induced by neglecting coupling can be very important. For a = - 30° and e/B = 0.1, rotation is positive with coupling and negative without coupling!

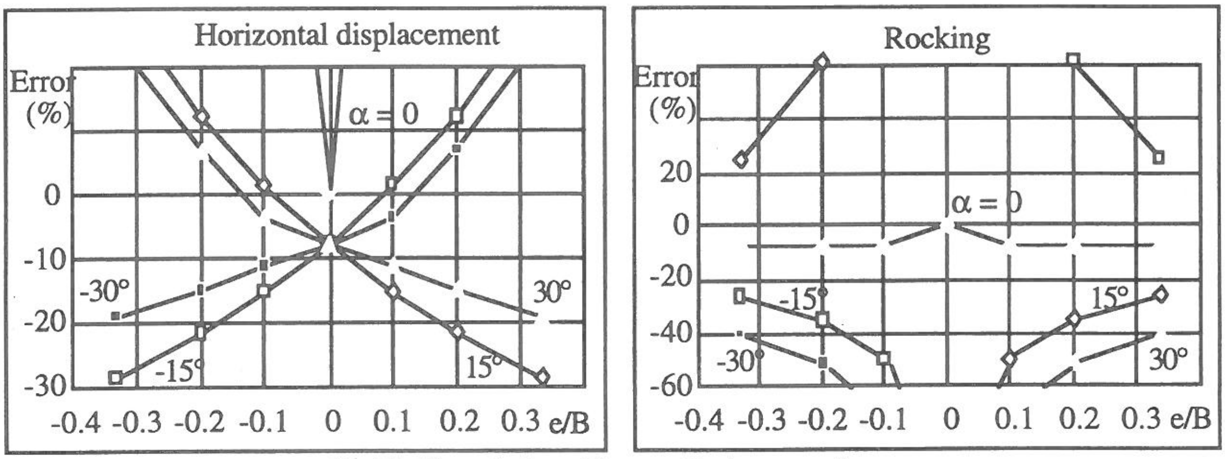

fig.11. Error induced by neglicting coupling

Therefore, the coupling between horizontal translation and rocking cannot be neglegt if the footing is embedded! Therefore, the coupling beetwen horizontal translation and rocking cannot be neglected if the footing is embedded! It is very important that authors give all results concerning the impedance matrix, and not only the terms of main diagonal as it is often the practice.

3.3 *Error induced by neglecting slope of substratum*

To encourage researchers to publish in this way, we give new results concerning the impedance functions of non embedded strip foundations on a layer resting on a rigid inclined substratum. The calculations are performed with the software SSI 2D/3D developed by Schmid (1988). For the reasons indicated previously, we give only here the results for :

- internal damping : $\beta = 5\%$,
- Poisson's ratio : $\nu = 0.3$,
- and layer thickness ratio H/B = 2,

in form of compliance functions (displacement functions) which are the inverse of the impedance functions. Particular case $\alpha = 0$ was also calculated by HUH (1986) with the same software, but in a previous version. The results are presented on fig. 12.

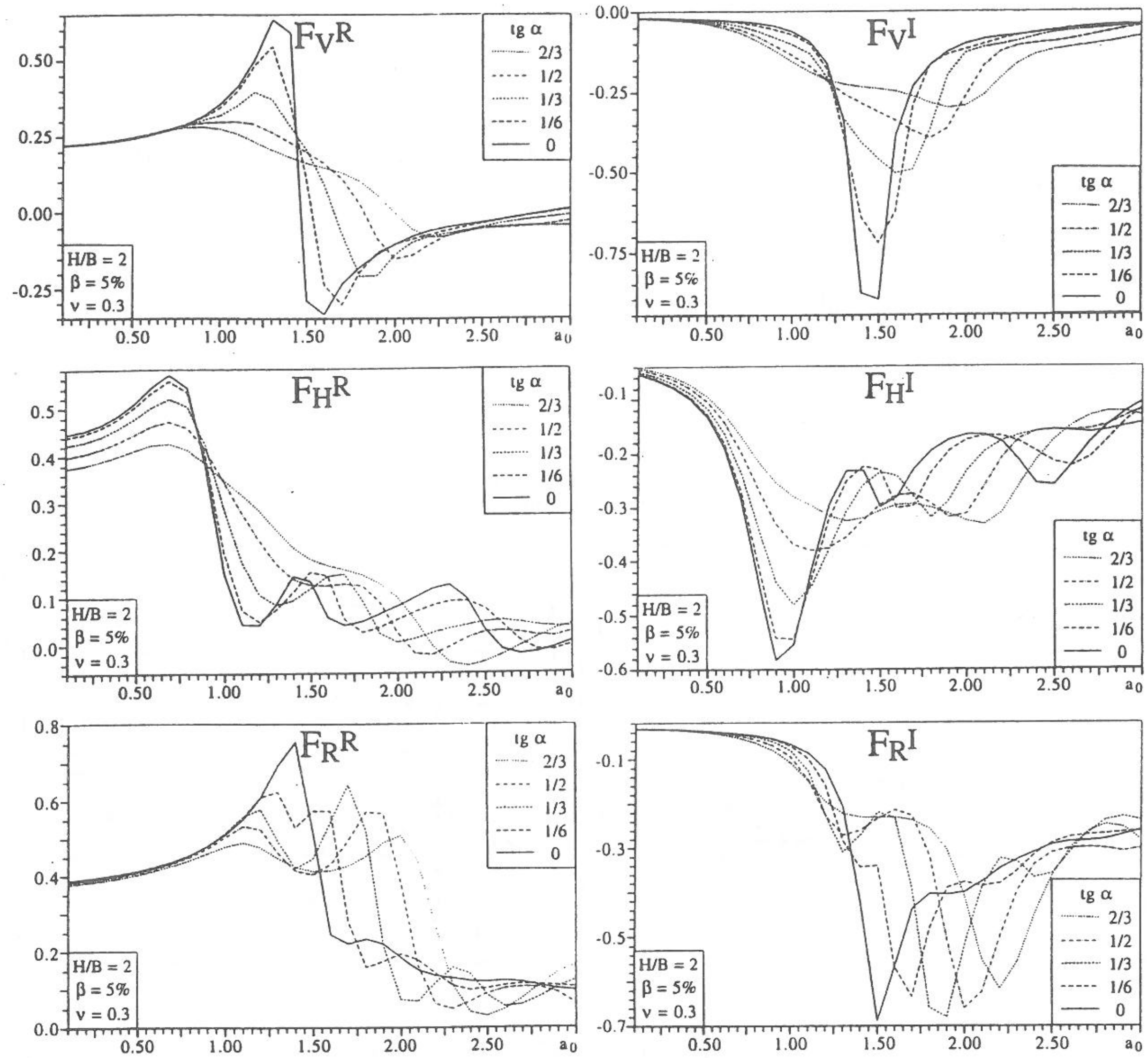

fig.12. Compliance functions (strip footing)

We can see that :

- vertical and rocking stiffnesses are not influenced by value of slope contrary to horizontal stiffness,
- all dynamic compliance functions are highly influenced by value of the slope for H/B = 2. This influence decreases with the increase of the layer thickness and can be neglecgt for H/B ≥ 10, at least for the rocking mode. It is clear that for great values of the ratio H/B, the behaviour of the layer is nearly that of the half-space medium so that the substratum slope have no significant influence.

It is interesting to estimate the differences induced by a slope on the amplitudes of horizontal translation and rocking movements due to an horizontal harmonic force. We have choosen the system described on fig. 8 according the values :

- width : 2B = 2 m
- thickness : H = 2 m
- mass of the system : M = 6,000 kg/ml
- inertial moment of the system about horizontal axis passing through the center of gravity: I = 4,000 kg m^2 /ml

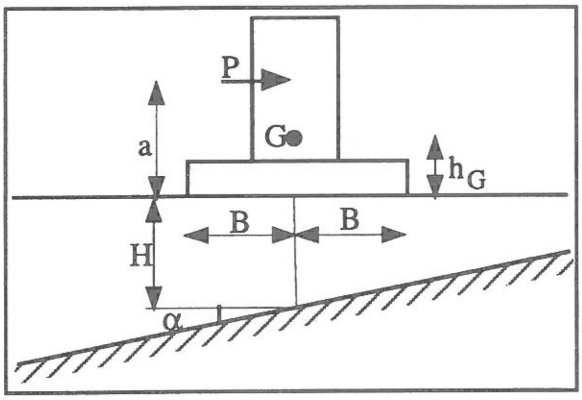

fig. 13. System loaded by an horizontal force

- distance from the center of gravity to the surface of soil : $h_G = 0.7$m
- distance from the application force to the surface of soil : $a = 1.5$ m
- shear modulus of the layer : $G = 45$ MPa
- Poisson's ratio of the layer : $\nu = 0.3$
- soil density : $\rho = 2{,}000$ kg m^{-3}
- slope of substratum : $\operatorname{tg} \alpha = 0$ or $1/3$
- circular frequency : $\omega = 150$ rd s^{-1}

These values correspond to a dimensionless circular frequency $a_0 = 1$.

Eigenfrequency values of a layer (horizontal substratum) can be calculated by using equations 5 and 6. We note respectively a_{eH} and a_{eV} the eigen frequencies corresponding to horizontal and vertical movements.

$$a_{eH} = \frac{\pi}{2}\frac{B}{H} = 0.785 \tag{5}$$

$$a_{eV} = \frac{\pi}{2}\frac{B}{H}\sqrt{\frac{2(1-\nu)}{1-2\nu}} = 1.469 \tag{6}$$

These frequencies do not correspond to the excitation frequency.

As the center of gravity of the system is not on the surface of the soil, we have automatically a coupling between horizontal translation and rocking. The impedance values are assessed at center of gravity of the soil-footing surface. To calculate the horizontal displacement u_H and the rotation θ about y-axis, we have to solve the equations :

$$\begin{gathered}(K_H - M\omega^2)\, u_H + K_H h_G\, \theta = |P| \\ K_H h_G\, u_H + (K_R + K_H h_G^2 - I\omega^2)\, \theta = -(a - h_G)\, |P|\end{gathered} \tag{7}$$

where $|P|$ is the amplitude of the external force P.

Table 2 gives the used values of compliance and impedance functions deduced from fig. 12.

Slope	$G F_H^R$	$G F_H^I$	$GB^2 F_R^R$	$GB^2 F_R^I$	K_H^R / G	K_H^I / G	K_R^R / GB^2	K_R^I / GB^2
$\operatorname{tg} \alpha = 0$	0.147	-0.553	0.510	-0.069	0.449	1.689	1.926	0.261
$\operatorname{tg} \alpha = 1/3$	0.287	-0.481	0.510	-0.084	0.915	1.533	1.909	0.314
difference (%)	64.5	13.9	0	19.6	68.3	9.7	0.9	18.4

Table 2. Values of compliance and impedance functions.

Slope	$\lvert u_H \rvert / \lvert P \rvert\ 10^{-8}$	$\lvert \theta \rvert / \lvert P \rvert\ 10^{-8}$
$\operatorname{tg} \alpha = 0$	1.356	2.005
$\operatorname{tg} \alpha = 1/3$	1.357	1.618
difference	<1 %	21.4 %

Table 3. Values of displacement and rotation.

Table 3 gives the amplitude $|u_H|$ and $|\theta|$ obtained by solving equations 7. It is interesting to note a very important difference on real part of the horizontal stiffness, and no difference on horizontal displacement. On the other hand, we have no significant difference on the real part of rocking stiffness, but an important difference on rocking amplitude!

This example shows that only a complete calculation can give precise indication on the influence of a parameter.

In the same way, we published recently results concerning the static stiffnesses given in an approximate maner (validity range : tg $\alpha < 1$, $1 \leq H/B \leq 12$, $\nu \geq 0.25$). We confirm that the static stiffness is not influenced by slope of substratum for vertical and rocking movements. Concerning horizontal movement, static stiffness is sensitive to substratum slope only for Poisson's ratio greater than 0.3.

4. CONCLUSION

In the background of this discussion the question - very important for all teachers - is also about contents of the education of our students. Dynamic soil-structure interaction can be teached :

- in giving practical methods sufficient for solve classical simple cases,
- in developing theoretical methods as Boundary Elements Method and in using performing software to have a through knowledge of these subjects. In this way, there is a risq that the education will emphases on Mathematics or Computers than on Civil Engineering materials.

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